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Design optimization of a spreader heat sink for power electronics

Gaël Maranzana [a,*], Isabelle Perry [a], Denis Maillet [a], Stephane Raël [b]

[a] *Laboratoire d'energétique et de mécanique théorique et appliquée UMR 7563 CNRS–INPL–UHP, 2, avenue de la forêt de Haye, BP 160, 54504 Vandoeuvre-lès-Nancy cedex, France*

[b] *Groupe de recherche en electrotechnique et electronique de Nancy, 2, avenue de la forêt de Haye, 54516 Vandoeuvre-lès-Nancy cedex, France*

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Abstract

This article deals with a design method for optimizing heat spreaders dedicated to electronic board cooling. The modeling is based on the thermal quadrupole method which is an analytical exact and rapid method that can be implemented for suitable geometries. Under some conditions, an optimal thickness can be found for the spreader (or the heat sink base). It correspond to a minimization of the average or the maximal temperature of the heat sources. This optimal thickness is given in an non-dimensional form by an abacus which can be used in a quantitative way to design heat spreaders or, in a more qualitative way, to assess the function and the performance of the spreader. Locations of the sources on the heat spreader as well as the shape of the spreader are also optimized. Finally, the case of a pyramidal multi-layer heat spreader is considered in order to test the efficiency of the quadrupole method as a tool for modeling conductive heat transfer for optimization applications.

Keywords: Heat spreader; Heat sink; Thermal quadrupole method; Power electronics; Design optimization; Thermal modeling; Laplace transform; Fourier cosine transform

1. Introduction

Cooling electronic devices [1,2] require the use of heat spreaders whose function is to allow the spreading of the flux lines in the 3D space and to increase the exchange area with the coolant. It is possible to model a spreader of the type shown in Fig. 1 by a sum of three thermal resistances (R_d, R_c and R_f) characterizing the non-disturbed spreader, the constriction of the flux lines and the convective transfer with the coolant respectively:

$$R_t = R_d + R_c + R_f \tag{1}$$

$$R_d = \frac{e}{\lambda L_0^2}, \quad R_f = \frac{1}{h L_0^2} \quad \text{(in the case of Fig. 1)} \tag{2}$$

the total resistance being defined with respect to a temperature difference between the average temperature of the source and the temperature of the coolant (which is set here at 0 °C). The calculation of the constriction resistance R_c can be done using different correlations found in the literature. For example Degiovanni et al. [3,7] suggest, for thick spreaders:

$$R_c = \frac{8}{3\pi^{3/2}\lambda L_e}\left(1 - 1.288\frac{L_e}{L_0} + 0.288\left(\frac{L_e}{L_0}\right)^{3.75}\right) \tag{3}$$

Under the same conditions, Negus et al. [4] propose:

$$R_c = \frac{0.475 - 0.62 L_e/L_0 + 0.13(L_e/L_0)^2}{\lambda L_e} \tag{4}$$

These correlations have been determined using cylindrical geometries but stay also valid in the case where the source and the spreader have nearly square bases. These resistances are defined with respect to the average temperature of the excited surface, considering a uniform excitation flux. This type of correlation is only valid for a thick heat spreader, whose thickness is of the same order of magnitude as its lateral lengths because they do not take into account the exchange with the coolant. These constriction resistances are not intrinsic to the spreader, they also depend on the convective transfer, or more globally on the boundary conditions. In fact, it is observed that the lower the convective transfer, the more significant the spreading of the flux lines.

In practice, spreaders are generally not thick enough for considering constriction resistances independent of the

* Corresponding author.
E-mail address: gmaranza@ensem.inpl-nancy.fr (G. Maranzana).

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Nomenclature

B_e	Biot number relative to the spreader thickness
$(B_e)_{\text{opt}}$	optimal Biot number relative to the spreader thickness
B_{Le}	Biot number relative to the source size
B_{L0}	Biot number relative to the spreader lateral dimensions
e	thickness of the spreader
h	heat transfer coefficient
L_e^x	source length in the x-direction
L_e^y	source length in the y-direction
L_0^x	spreader length (x-direction)
L_0^y	spreader length (y-direction)
R_c	constriction resistance
R_d	non-disturbed spreader resistance
R_f	convective heat transfer resistance
R_t	total resistance
T	temperature
T_{1D}	temperature without spreader
$x_p,\ y_p$	location of the p-th source
α_n	x-eigenvalue of order n
β_m	y-eigenvalue of order m
$\gamma_{n,m}$	$=\sqrt{\alpha_n^2+\beta_m^2}$
$\theta_{n,m}$	harmonic of temperature, of order (n,m)
$\phi_{n,m}$	harmonic of flux density, of order (n,m)
λ	conductivity of the spreader
Ω, Ψ	functions
φ	flux density dissipated by the source

Subscripts or superscripts

p	relative to source number p
max	relative to the maximal temperature
ave	relative to the average temperature
opt	optimal value

boundary conditions, so Yovanovich and Antonetti [5] have established an abacus giving the constriction resistance for a spreader of such a thickness with an imposed temperature on the exchange surface. Song et al. [6] propose a correlation that integrates the convective coefficient and the thickness of the spreader as well:

$$R_c = \frac{(1-L_e/L_0)^{3/2}}{2\lambda L_e} \times \frac{(\lambda/h)(\pi^{3/2}/L_0+1/L_e)+\tanh((\pi^{3/2}/L_0+1/L_e)e)}{1+\frac{\lambda}{h}(\pi^{3/2}/L_0+1/L_e)\tanh((\pi^{3/2}/L_0+1/L_e)e)} \tag{5}$$

According to its authors, this correlation is valid in almost all the common applications in electronics with a 10% maximum error. One can notice that resistance R_c is a decreasing function of the thickness of the spreader.

The total resistance of the spreader shown in Fig. 1 is plotted versus its shape ratio for the different correlations as well as the exact model defined further in this article (Eq. (21)) in Fig. 2 ($h = 1000\ \text{W·m}^{-2}\text{·K}^{-1}$, $\lambda = 400\ \text{W·m}^{-1}\text{·K}^{-1}$, $L_0 = 100$ mm, $L_e = 20$ mm). Degiovanni's and Negus' correlation give the same results which are valid for this particular example where the thickness is larger than the half of the lateral length of the spreader. Song's correlation is valid on the whole thickness domain within a certain accuracy. We can already remark that an optimal thickness that minimizes the total thermal resistance exists.

This article displays a methodology for calculating the optimal dimensions of a spreader. It is aimed at answering the following question: knowing the dimensions of the sources, the dissipated power, the conductivity of the spreader material, as well as the convection coefficient, what

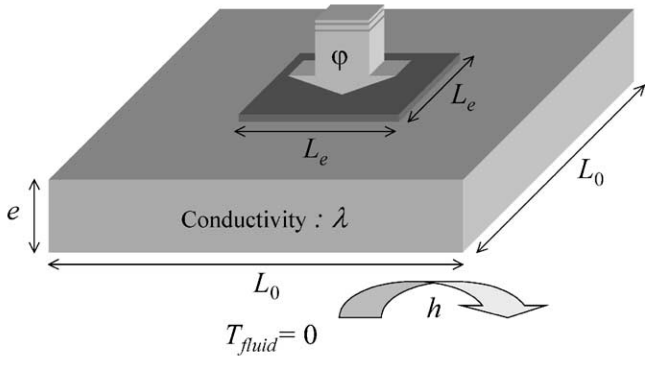

Fig. 1. Single layer square base spreader.

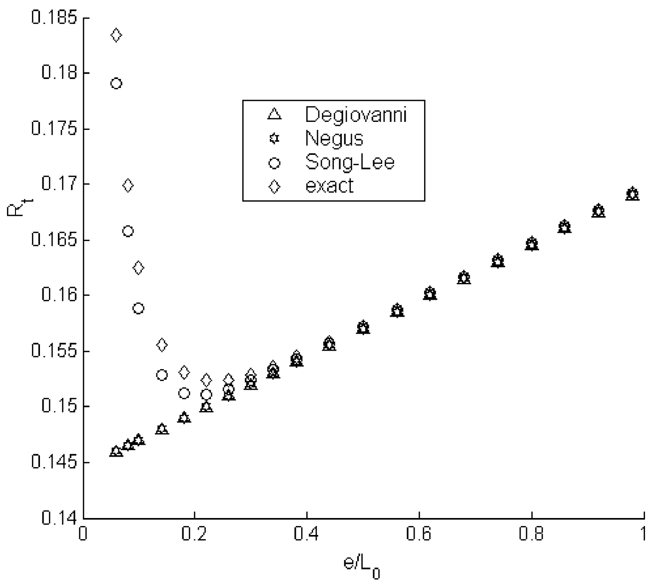

Fig. 2. Total thermal resistance of the system as a function of the thickness of the spreader, using different correlations for the constriction resistance and an exact method.

are the dimensions of the spreader and the locations of the sources that minimize their temperatures?

As shown in the preceding example, an optimum exists for the thickness of the spreader. But an optimum does not exist for its base surface area because the larger its value, the larger the exchange surface and the more important the spreading of the flux lines. So, an extra criterion should be introduced, to determine this area. It could be the maximal allowable temperature of the source.

2. Case of a single layer heat spreader

2.1. General case

Let us consider a single layer heat spreader whose aim is the cooling of a set of P sources as shown in Fig. 3. Its lateral lengths are called L_0^x and L_0^y, its thickness e and its conductivity λ. The lower face of the spreader is submitted to a uniform convective heat transfer coefficient h and the lateral faces are insulated. Each source number p, of lengths L_p^x and L_p^y in the x and y-directions respectively dissipates a known flux density $\varphi_p(x, y)$ on its heating surface.

Any increase in the thickness of the spreader yields a better spreading of the flux lines (a decrease of R_c) but also leads to a simultaneous increase in the non-disturbed resistance of the heat spreader ($R_d = e/\lambda L_0^x L_0^y$). So, an optimal thickness that minimizes the temperature of the sources, clearly exists. As in the previous single source case, no optimum on the lateral dimensions of the spreader (L_0^y, L_0^x) exists. However, if the base surface area $L_0^y \times L_0^x$ is fixed, an optimum exists for the shape of the spreader, that is to say for the L_0^y/L_0^x ratio. In this latter case one or several optima can be found for the locations of the sources ($[x_p, y_p]$).

Thus, the aim of this section is to optimize the thickness of the spreader (e), its shape (L_0^y/L_0^x) and the locations of the sources ($[x_p, y_p]$) as a function of some control parameters which are the spreader base surface area ($L_0^y \times L_0^x$), the dissipated flux densities ($\varphi_p(x, y)$), the convective heat transfer coefficient (h), and the conductivity (λ) of the material the spreader is made out of.

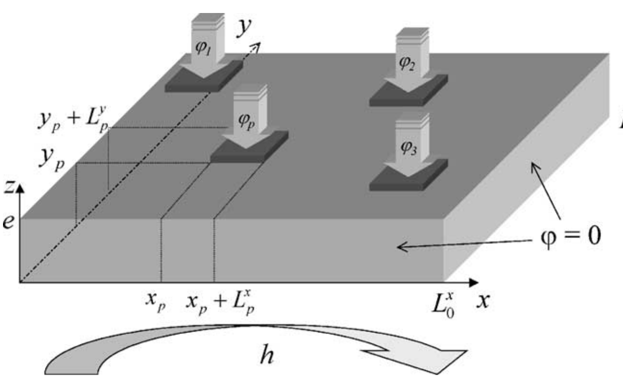

Fig. 3. Single layer spreader: general case.

The thermal quadrupole method [7] (based on integral transforms and on the method of separation of variables) yields the analytical expression of temperature T of the upper face of the spreader as a function of the imposed flux density.

For steady state conditions, the Fourier-cosine transform of the temperature T is written:

$$\theta_{0,0} = \left(\frac{e}{\lambda} + \frac{1}{h}\right)\phi_{0,0} \tag{6}$$

$$\theta_{n,m} = \frac{1}{h}\frac{1 + (h/\lambda\gamma_{n,m})\tanh(\gamma_{n,m}e)}{1 + (\lambda\gamma_{n,m}/h)\tanh(\gamma_{n,m}e)}\phi_{n,m} \quad \text{for } (n, m) \neq (0, 0) \tag{7}$$

where $\theta_{n,m}$ and $\phi_{n,m}$ are the Fourier-cosine transforms of temperature T and of the flux density φ on the upper face of the spreader ($z = e$):

$$\theta_{n,m} = \int_0^{L_0^x}\int_0^{L_0^y} T(x, y, z = e)\cos(\alpha_n x)\cos(\beta_m y)\,\mathrm{d}y\,\mathrm{d}x$$

$$\phi_{n,m} = \int_0^{L_0^x}\int_0^{L_0^y} \varphi(x, y, z = e)\cos(\alpha_n x)\cos(\beta_m y)\,\mathrm{d}y\,\mathrm{d}x \tag{8}$$

with

$$\alpha_n = \frac{n\pi}{L_0^x}, \qquad \beta_m = \frac{m\pi}{L_0^y}, \qquad \gamma_{n,m} = \sqrt{\alpha_n^2 + \beta_m^2} \tag{9}$$

So, the spectrum of the imposed flux density on the upper face of the spreader is given by:

$$\phi_{n,m} = \sum_{p=1}^{P}\int_{x_p}^{x_p+L_p^x}\int_{y_p}^{y_p+L_p^y} \varphi_p(x, y)\cos(\alpha_n x)\cos(\beta_m y)\,\mathrm{d}y\,\mathrm{d}x \tag{10}$$

As a result, the analytical value of the temperature spectrum on the upper face of the spreader can be calculated. Knowing this analytical spectrum, the real value of temperature can be found using either a fast Fourier inverse transform or keeping the analytical character of the method:

$$T(x, y, z = e) = \frac{\theta_{0,0}}{L_0^x L_0^y} + \frac{2}{L_0^x L_0^y}\sum_{n=0}^{N}\sum_{m=0}^{M}(2 - \delta_{n,0} - \delta_{0,m}) \times \cos(\alpha_n x)\cos(\beta_m y)\theta_{n,m} \tag{11}$$

where N and M are the truncation orders, in the x- and y-directions of the Fourier series (chosen according to the spatial resolution desired [9]), and δ the Kronecker symbol: $\delta_{i,j} = 1$ if $i = j$ and $\delta_{i,j} = 0$ if $i \neq j$.

If we make the assumption that the sources are isothermal in the thickness direction, two natural objective functions can be introduced: the maximum of the average temperatures of the sources and the maximum local temperature:

$$T_{\text{ave}} = \max_{p}\left(\frac{1}{L_p^x L_p^y}\int_{xp}^{xp+L_p^x}\int_{yp}^{yp+L_p^y} T(x,y,e)\,\mathrm{d}x\,\mathrm{d}y\right)$$
$$\text{or } T_{\max} = \max_{x,y}\bigl(T(x,y,e)\bigr) \qquad (12)$$

We choose here to keep the analytical character of the method in order to get an analytical expression for the objective function and for its gradient, which allows the use of an efficient minimization method.

2.2. Particular cases

The aim of this section is to allow a functional analysis of the heat spreader by considering some basic examples.

2.2.1. Design of a heat spreader for a single source

Here, we introduce a procedure for an optimal design of a heat spreader used to cool a square base heat source ($L_e^x = L_e^y = L_e$) that dissipates a uniform heat flux (case of Fig. 1). The optimal location of the source is of course the center of the spreader. According to the symmetry, only the quarter of the structure requires to be modeled and the optimal shape of the spreader being square the optimal length ratio $r = L_0^Y / L_0^x$ is equal to unity ($L_0^x = L_0^y = L_0$). The size of the source L_e, the uniform dissipated flux φ, the convective transfer coefficient h and the conductivity of the spreader λ are assumed to be known. In this problem the optimization concerns only one variable, the thickness of the spreader (e), the adjustment parameter being the area of the lower face (L_0). Let us calculate first the objective function.

The first step is to calculate the flux density spectrum imposed on the upper face of the spreader using Eq. (10):

$$\phi_{n,m} = \varphi \int_0^{L_e/2}\int_0^{L_e/2} \cos(2\alpha_n x)\cos(2\beta_m y)\,\mathrm{d}y\,\mathrm{d}x \qquad (13)$$

since only the quarter of the structure is modeled. (L_0 and L_e are replaced by $L_0/2$ and $L_e/2$ here.) So,

$$\phi_{0,0} = \varphi\left(\frac{L_e}{2}\right)^2 \qquad (14)$$

$$\phi_{\substack{0,m\\(m\geqslant 1)}} = \varphi \frac{L_e}{2}\frac{\sin(\beta_m L_e)}{2\beta_m}$$

$$\phi_{\substack{n,0\\(n\geqslant 1)}} = \varphi \frac{L_e}{2}\frac{\sin(\alpha_n L_e)}{2\alpha_n}$$

$$\phi_{\substack{n,m\\(n,m\geqslant 1)}} = \varphi \frac{\sin(\alpha_n L_e)}{2\alpha_n}\frac{\sin(\beta_m L_e)}{2\beta_m} \qquad (15)$$

The two objective functions (12) can be considered (minimization of the average or of the maximum local source temperature):

$$T_{\text{ave}} = \frac{4}{L_e^2}\int_0^{L_e/2}\int_0^{L_e/2} T(x,y,e)\,\mathrm{d}y\,\mathrm{d}x \quad \text{or}$$
$$T_{\max} = \max_{x,y\leqslant L_e/2}\bigl(T(x,y,e)\bigr) \qquad (16)$$

The objective function is the average temperature of the source

Eqs. (6), (7), (11) and (14)–(16) yield:

$$T_{\text{ave}} = \frac{\varphi L_e^2}{L_0^2}\left(\frac{e}{\lambda} + \frac{1}{h}\right) + \frac{\varphi L_e}{\lambda}\sum_{n=0}^{N}\sum_{m=0}^{M} a_{n,m}^{\text{ave}} \times \frac{2\lambda\gamma_{n,m}/h + \tanh(2\gamma_{n,m}e)}{1 + (2\lambda\gamma_{n,m}/h)\tanh(2\gamma_{n,m}e)} \qquad (17)$$

with

$$a_{n,m}^{\text{ave}}(L_e/L_0) = \frac{L_0}{\pi^3 L_e}\Biggl[\frac{2L_0^2/L_e^2(1-(\delta_{n,0}+\delta_{0,m})+\delta_{n,0}\delta_{0,m})}{n^2 m^2 \pi^2\sqrt{n^2+m^2}} \times \sin^2\left(n\pi\frac{L_e}{L_0}\right)\sin^2\left(m\pi\frac{L_e}{L_0}\right) + \frac{(\delta_{0,m}-\delta_{n,0}\delta_{0,m})}{n^3}\sin^2\left(n\pi\frac{L_e}{L_0}\right) + \frac{(\delta_{n,0}-\delta_{n,0}\delta_{0,m})}{m^3}\sin^2\left(m\pi\frac{L_e}{L_0}\right)\Biggr] \qquad (18)$$

Remark. The equivalent total thermal resistance of the system is:

$$R_t = \frac{T_{\text{ave}}}{\varphi L_e^2} \quad (T_{\text{fluid}} = 0) \qquad (19)$$

So, if the resistances of the non-disturbed spreader and of the convective transfer are subtracted from the constriction resistance, the following result is obtained:

$$R_c = R_t - \frac{1}{hL_0^2} - \frac{e}{\lambda L_0^2} \qquad (20)$$

Comparing Eqs. (17), (18) and (20) yields:

$$R_c(h,\lambda,L_e,L_0) = \frac{1}{\lambda L_e}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} a_{n,m}^{\text{ave}} \times \frac{2\lambda\gamma_{n,m}/h + \tanh(2\gamma_{n,m}e)}{1 + (2\lambda\gamma_{n,m}/h)\tanh(2\gamma_{n,m}e)} \qquad (21)$$

which is the exact analytical expression of the constriction resistance (based on the average temperature). Its approximation is Song's correlation (5). This resistance always depends on the convective heat transfer coefficient, which confirms that the constriction resistance is not an intrinsic property of the spreader since it depends on the boundary conditions (h). When the thickness of the spreader becomes large enough when compared with its lateral lengths (Degiovanni's [3] and Negus' [4] assumption), the $\tanh(2\gamma_{n,m}e)$ terms (in (21)) go to unity, and the constriction resistance R_c does not depend on the convective coefficient anymore:

$$R_c(\lambda,L_e,L_0) = \frac{1}{\lambda L_e}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} a_{n,m}^{\text{ave}}\left(\frac{L_e}{L_0}\right) \qquad (22)$$

In order to provide a more general character to this study, the objective function can be given a non-dimensional form.

The aim of the spreader is to reduce the temperature with respect to a reference configuration where heat transfer is one-dimensional. So the maximum temperature the source can reach corresponds to a cooling without spreader:

$$T_{1D} = \frac{\varphi}{h} \tag{23}$$

The objective function is normalized with this quantity:

$$T^*_{\text{ave}} = \frac{T_{\text{ave}}}{T_{1D}} \tag{24}$$

The non-dimensional parameters of the optimization problems are introduced in (17), (18), (23), (24) with:

- $B_{L0} = hL_0/(2\lambda)$: the Biot number relative to the lateral lengths of the spreader.
- $B_e = he/(2\lambda)$: the Biot number relative to the thickness of the spreader.
- $B_{Le} = hL_e/(2\lambda)$: the Biot number relative to size of the source.

So the objective function depends on three parameters and can be written under a functional form:

$$T^*_{\text{ave}} = \Omega_{\text{ave}}\left(\frac{L_0}{L_e}, B_{Le}, B_e\right) \tag{25}$$

Using a symbolic calculus software of the Maple® type, the analytical value of the gradient of the objective function $(\partial T^*_{\text{ave}}/\partial B_e)$ can be determined and an efficient minimization method can be implemented to calculate the optimal Biot number relative to thickness $(B_e)_{\text{opt}}$ for different values of B_{Le} and L_0/L_e.

The results are presented under the form of an abacus allowing the optimal choice of the thickness of the spreader–see Fig 4(a). This abacus can be represented by a function Ψ_{ave}:

$$\frac{\partial \Omega_{\text{ave}}}{\partial B_e}\left(\frac{L_0}{L_e}, B_{Le}, (B_e)_{\text{opt}}\right) = 0 \tag{26}$$

or

$$(B_e)_{\text{opt}} = \Psi_{\text{ave}}\left(\frac{L_0}{L_e}, B_{Le}\right) \tag{27}$$

The average non-dimensional temperature for this optimal thickness can also be calculated to allow the choice of the lateral lengths of the spreader to get an optimum for the spreader dimensions: see Fig. 4(b). This abacus can be represented by a function $\Omega^{\text{opt}}_{\text{ave}}$:

$$\frac{hT^{\text{opt}}_{\text{ave}}}{\varphi} = \Omega_{\text{ave}}\left(\frac{L_0}{L_e}, B_{Le}, (B_e)_{\text{opt}}\right) = \Omega^{\text{opt}}_{\text{ave}}\left(\frac{L_0}{L_e}, B_{Le}\right) \tag{28}$$

where Ω_{ave} is defined by Eq. (25).

So, abacus 4(b) yield the lateral lengths of the spreader as a function of the maximum allowable average temperature while abacus 4(a) yield the corresponding optimal thickness.

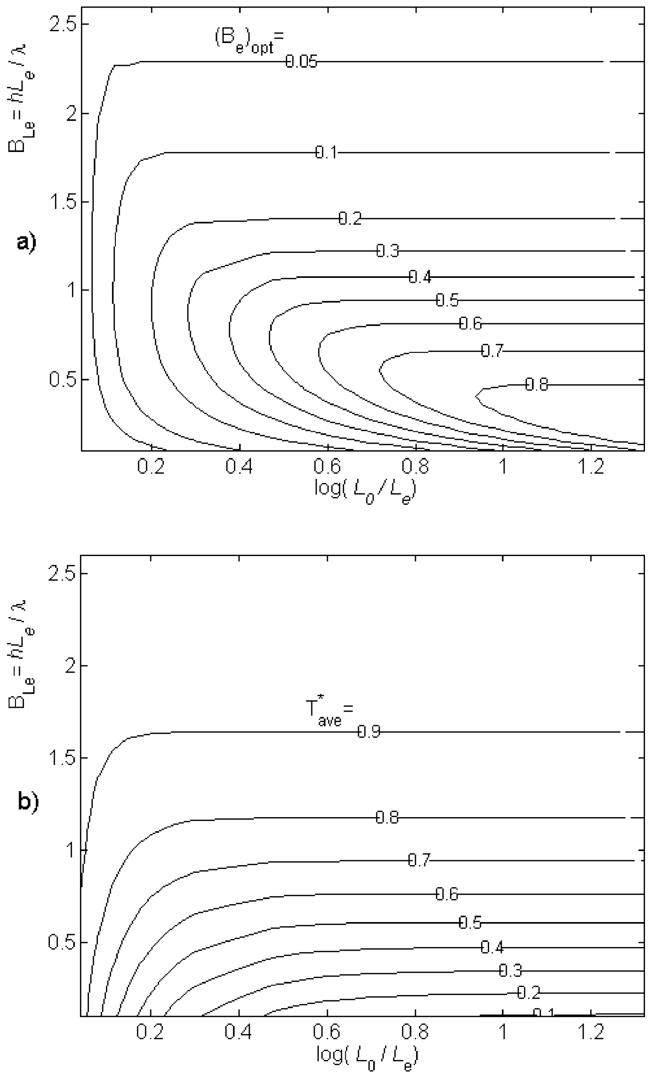

Fig. 4. (a) Optimal non-dimensional thickness (he/λ) that minimizes the average temperature of the source as a function of the non-dimensional length of the source (hL_e/λ) and of the length ratio (L_0/L_e): function Ψ_{ave}. (b) Non-dimensional average temperature for an optimal non-dimensional thickness: function $\Omega^{\text{opt}}_{\text{ave}}$.

Interpretations. Even if the underlying interpretations are related to the particular case of the cooling of a single source by a single-layer spreader, they can be extended to more general cases in a qualitative way.

• The non-dimensional optimal thickness of a spreader is an increasing function of its non-dimensional lateral lengths, see Fig. 4(a) (progression along a horizontal line, B_{Le} beeing kept constant). However this function reaches a plateau, where an increase of the spreader lateral lengths has a low impact on the source temperature (see Fig. 4(b)). This plateau is reached for low spreader dimensions if the Biot number relative to the size of the source $(B_{Le} = hL_e/2\lambda)$ is large. As a consequence, there exists a lateral length of the spreader, or exchange surface, that is not interesting to exceed.

- For a fixed value of the L_0/L_e ratio, an increase of Biot number B_{Le} (or B_{L0}) leads first to an increase of the optimal thickness $(B_e)_{\text{opt}}$ which correspond to a spreading of the flux lines. This increase in $(B_e)_{\text{opt}}$ is followed then by a decrease until it reaches a zero value (see Fig. 4(a), progression along a vertical line). This behavior is explained now. The optimal thickness can not increase indefinitely because the resistance of the non-disturbed spreader R_d would become too important. This optimal thickness being limited, the larger the Biot numbers B_{Le} (or B_{L0}), the more one-dimensional the heat transfer, particularly at the center of the source. At that point it can be said that heat transfer become *locally* one-dimensional. According to this, a spreader is not required in this type of locally one-dimensional situation.

The non-dimensional optimal thickness of a heat spreader is not a strictly increasing function of the non-dimensional size of the source; above a given Biot number (relative to the size of the source) it is better to decrease the thickness of the spreader than to increase it. If the exchange (h) is very good, the optimal thickness may be equal to zero, so it is better not to use any spreader.

- For all cases, the optimal Biot number relative to the thickness is lower than unity, so it would be a nonsense to design a spreader whose Biot number (he/λ) would be higher than one.

The objective function is the maximal temperature of the source

This type of objective function is less used than the previous one because its value can not be experimentally obtained by an electric measurement. Nevertheless, it corresponds to the maximal temperature (hot point) which is responsible for components break down.

$$T_{\max} = \max_{x,y \leqslant L_0/2} \big(T(x, y, z = e)\big) = T(0, 0, e) \tag{29}$$

since temperature is maximal at the center of the source, that is to say at $(x = 0,\ y = 0)$ when only one quarter of the structure is modelled.

The constriction resistance, based on the maximal temperature becomes:

$$R_c = \frac{1}{\lambda L_e} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} a_{n,m}^{max} \frac{2\lambda\gamma_{n,m}/h + \tanh(2\gamma_{n,m}e)}{1 + (2\lambda\gamma_{n,m}/h)\tanh(2\gamma_{n,m}e)} \tag{30}$$

with

$$\begin{aligned} a_{n,m}^{\max}\left(\frac{L_e}{L_0}\right) &= \frac{2L_e/L_0(1-(\delta_{n,0}+\delta_{0,m})+\delta_{n,0}\delta_{0,m})}{\pi^2 nm\sqrt{n^2+m^2}} \\ &\quad \times \sin\left(\frac{n\pi L_e}{L_0}\right)\sin\left(\frac{m\pi L_e}{L_0}\right) \\ &\quad + \frac{(\delta_{0,m}-\delta_{n,0}\delta_{0,m})}{\pi^2 n^2}\sin\left(\frac{n\pi L_e}{L_0}\right) \\ &\quad + \frac{(\delta_{n,0}-\delta_{n,0}\delta_{0,m})}{\pi^2 m^2}\sin\left(\frac{m\pi L_e}{L_0}\right) \end{aligned} \tag{31}$$

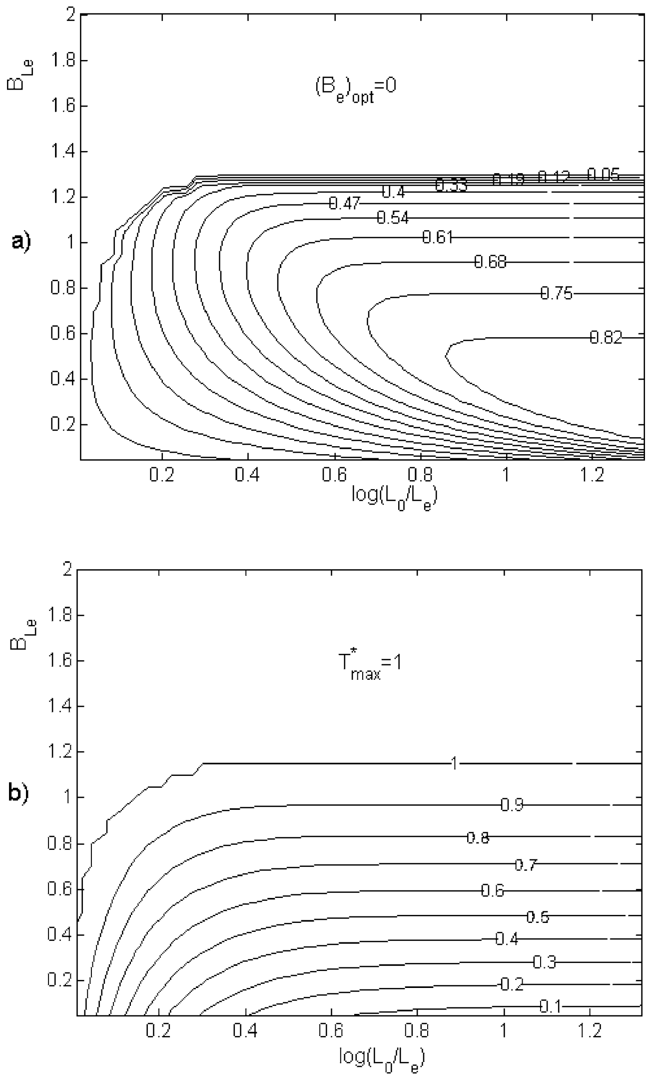

Fig. 5. (a) Optimal non-dimensional thickness (he/λ) that minimizes the maximum temperature of the source as a function of the non-dimensional length of the source (hL_e/λ) and of the length ratio (L_0/L_e): function $\Psi_{\max}$. (b) Non-dimensional maximum temperature for an optimal non-dimensional thickness: function $\Omega_{\max}^{\text{opt}}$.

In the same way as previously, the non-dimensional objective function is written as a function of three variables:

$$T_{\max}^* = \Omega_{\max}\left(\frac{L_0}{L_e}, B_e, B_{Le}\right) \tag{32}$$

The corresponding abacus are drawn in Fig. 5. Above a Biot number relative to the size of the source (B_{Le}) equal to 1.3 no optimal thickness exists anymore for a minimization of the maximum temperature of the source. However it has been shown previously that an optimal thickness that minimizes the average temperature of the source always exists (see Fig. 4).

It is due to the fact that the larger the Biot number (relative to the size of the source), the more one-dimensional the local transfer at the center of the source (no spreading) where the temperature reaches its maximum. However heat transfer in the neighborhood of the lateral sides of the source

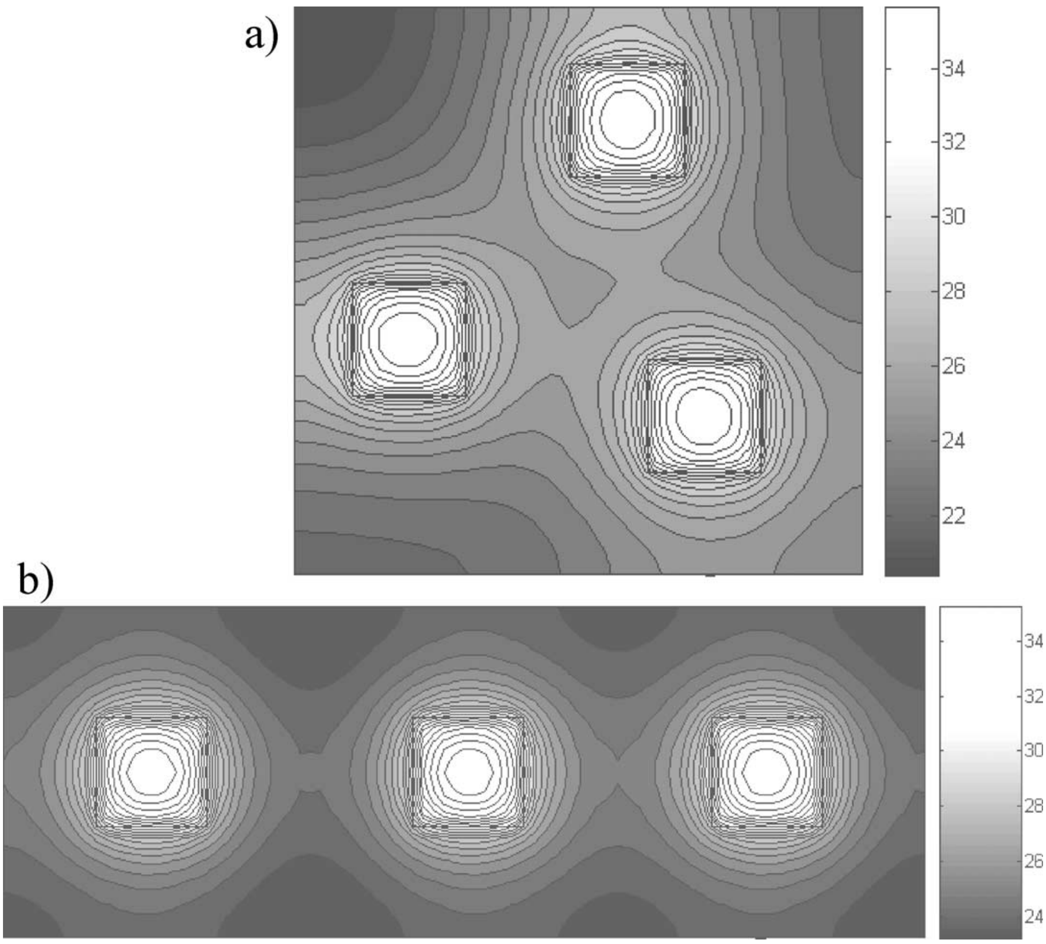

Fig. 6. (a) Temperature field (°C) on the upper face of the spreader after optimizing the thickness of the spreader and the location of the sources. (b) Temperature field (°C) on the upper face of the spreader, after optimizing the thickness of the spreader, its shape and the location of the sources.

keeps its three-dimensional character (spreading), which explains why the average temperature of the source longer takes advantage of the spreading of the flux lines.

2.3. Optimization of a heat spreader for three sources

Let us now consider the problem of the cooling of three sources (square base) of 1 cm^2 area each dissipating a 100 W uniform flux. A square heat sink of $5 \times 5 = 25$ cm^2 area is used. Its lower face is subjected to a uniform convective heat transfer coefficient $h = 5000$ $W \cdot m^{-2} \cdot K^{-1}$ (see [9] for considering a non-uniform heat transfer coefficient). The geometry is shown in Fig. 3 with $P = 3$. The objective function is the maximal temperature of the upper face of the heat spreader, and it is desired to optimize the location of the sources $([x_p, y_p])$ as well as the thickness of the spreader (e). So the problem requires an optimization in a *seven* parameters domain.

Direct application of the method described in Section 2.1 is conjugated with a minimization algorithm of the type of a sequential quadratic method (use of the function *fmincon* [8] in Matlab®), which performs global minimization, without analytical calculation of the gradient. It yields the result shown in Fig. 6(a). The solution is not unique because of the geometry symmetries. The result presented here has required 15 seconds of calculation for a 700 Mhz PC, choosing the initial parameter values:

$$[e = 5 \text{ mm};\ x_1 = x_2 = x_3 = y_1 = y_2 = y_3 = 20 \text{ mm}] \qquad (33)$$

The result of the optimization is:

$$[e = 6,9;\ x_1 = 24,3;\ x_2 = 5,0;\ x_3 = 31.0;\ y_1 = 35,0;\ y_2 = 15,7;\ y_3 = 9,0]\ (\text{mm}) \qquad (34)$$

Optimized locations of the sources are symmetrical with respect to the spreader diagonal. If this property had been taken into account from start, the number of parameters to be calculated would have been reduced to three.

Now, if it is desired to optimize the shape of the spreader, imposing a constraint on its base surface area ($L_x \times L_y = 25$ cm^2). The problem requires *nine* parameters with a non-linear constraint. The optimal geometry shown in Fig. 6(b) requires about two minutes of calculation to be obtained. It corresponds to $L_x = 3L_y$: each source is located at the center of a square whose surface area is the third of the whole surface area. This obvious result validates the method and underlines the good performance of the thermal quadrupole method for resolving such types of multiparameters optimization problems.

3. Case of a pyramidal heat spreader

In a more general way, the precedent study can be easily generalized to the case of multilayer heat spreaders, the thermal quadrupole method being able to deal with this kind of structures. The expression of the temperature of the sources stays explicit [7] when the layers have the same lateral lengths and becomes semi-analytical [9] when the stack is pyramidal (see the example shown in Fig. 7). For this last case, no analytical value for the temperature and its gradient can be easily found, but the quickness of calculus allowed by the thermal quadrupole method allows the optimization. For example, let us consider the pyramidal structure shown in Fig. 7. All the layers are square, the DBC (direct bounded copper) of 7.5 mm side is composed of two 0.3 mm thick copper ($\lambda = 380\ \mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) layers separated by an inner 1 mm thick AlN (Aluminum nitride: $\lambda = 170\ \mathrm{W{\cdot}m^{-1}{\cdot}K^{-1}}$) layer. The 5 mm length die is assumed to be isothermal in its thickness direction and is modeled by a source of 200 $\mathrm{W{\cdot}cm^{-2}}$ uniform flux density. A uniform convective heat transfer coefficient h equal to 3000 $\mathrm{W{\cdot}m^{2}{\cdot}K^{-1}}$ is considered on the lower face with a cooling fluid temperature of 0 °C. The heat sink base whose thickness has to be optimized is made of copper.

The thermal quadrupole method in the case of a pyramidal structure [9] allows to calculate the maximum temperature of the die. The minimization method is based on a SQP method (Sequential Quadratic Programming), without taking into account any analytical values for the maximum temperature or for the maximum temperature gradient. So, the optimal thickness of the heat sink can be plotted as a function of its lateral lengths—see Fig. 8(a)—and the corresponding maximal temperature too, see Fig. 8(b).

We have also made the corresponding optimization in the case of a single-bloc spreader with the flux dissipated by the die uniformly distributed on the upper face of the heat sink replacing the DBC. The corresponding optimal thickness is also plotted in Fig. 8(a). We can notice for this classical example that replacing the DBC and the die by a uniform flux density boundary condition does not change the optimal thickness much. Even if the flux density at the heat sink/DBC interface is not uniform—see Fig. 9, results on the optimal thickness stay roughly valid for this particular but common example.

Thus, the preceding studies and abaci can be used to design a heat sink for a component whose the structure is unknown, as a first approximation.

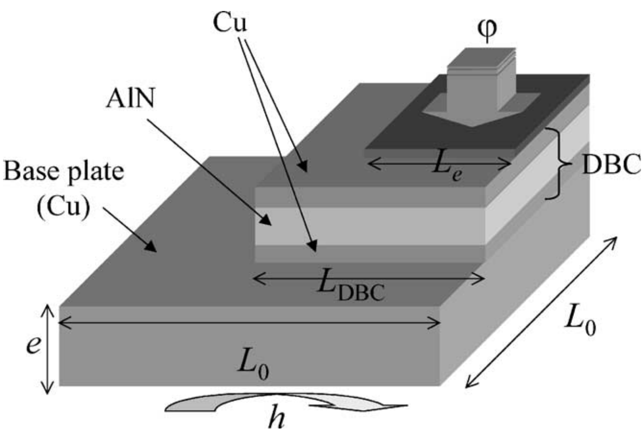

Fig. 7. A typical pyramidal heat spreader.

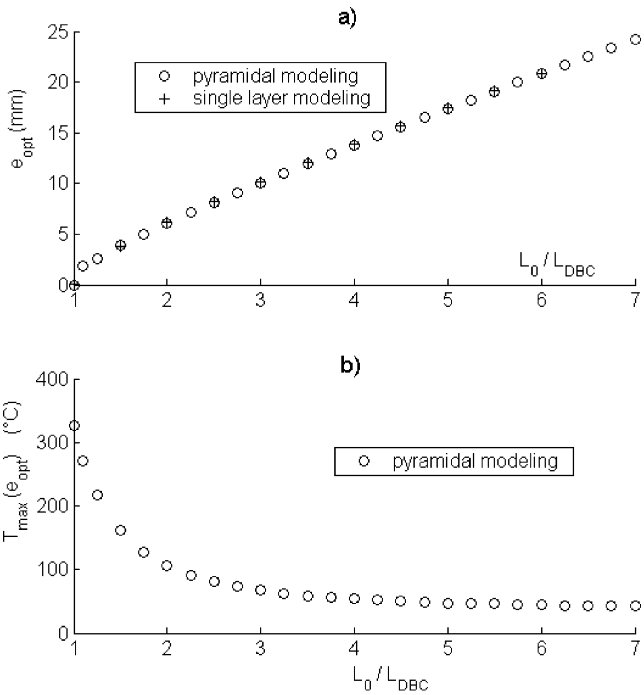

Fig. 8. (a) Optimal thickness of the heat sink as a function of its lateral lengths, for a pyramidal modeling and its equivalent single layer modeling. (b) Maximum temperature of the die for an optimal thickness.

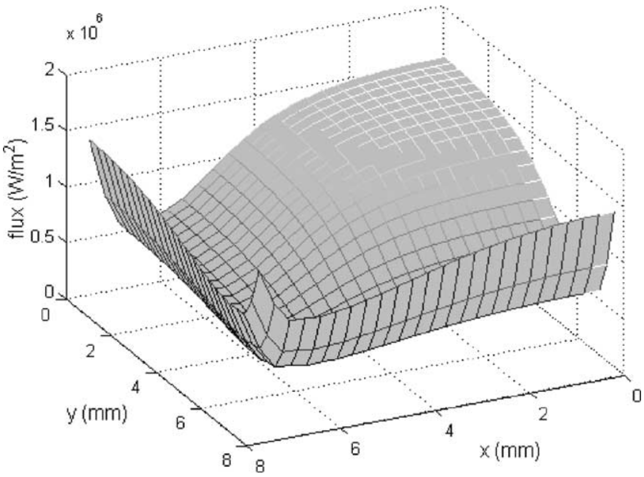

Fig. 9. Flux density on the lower face of the DBC for $L_0 = 2L_{DBC}$.

One can notice here that the sensitivity of the maximum temperature to the lateral lengths of the heat sink at the optimum thickness is very high when the exchange surface area is low. In fact, the qualitative interpretations are the same in the pyramidal case and in the single layer spreader case.

4. Conclusion

The optimal design of heat spreaders for electronics cooling implies the knowledge of the working conditions (dissipated power distribution, convective heat transfer coefficient) as well as the internal geometry of the system. If the lateral lengths of the spreader are fixed, an optimal thickness that minimizes the maximal or average temperature exists in usual working conditions. Abaci that permit the optimal design of a single layer heat spreader have been constructed.

The assumption of uniform flux density on the component base plate allows, as a first approximation, to choose the quasi-optimal thickness of an heat sink base whose lateral lengths are known.

The thermal quadrupole method is well suited to thermal optimization of electronic components. Very short processing times allow a design optimization which does not require the use of reduced models that may lead to bias because of oversimplification of the transfer problem. The method is limited to linear problems (there are some exceptions) and to structures made of stacks of parallelepipedic blocks. Future prospects will concern the optimization of heat exchangers by coupling the conductive model with a convective one.

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